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Vortex identification for the study of atmospheric flow: a new vortex criterion applied to wind resource assessment

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Abstract – Vortex identification is a non-consensual point of discussion among fluid mechanics researchers. Classical criteria such as Q and Δ are hereby confronted with a new definition.
A pertinent task for wind power project development studies is the post-processing analysis of the wind resource simulations in order to improve the project design. There is a large interest in extracting more information from typical commercial software – like WindSim, OpenWind, Meteodyn, WAsP – outputs at a minimal cost.
The proposed methodology of vortex identification in flows contributes with a more sophisticated wind resource analysis. It is also interesting to note that the additional financial investment and computational effort to perform this stage's calculations is negligible when compared to the previous calculation steps.

Introduction

Vortex, as an entity, is not consensually defined in the literature, being a point of discussion among fluid mechanic's researchers. Classical criteria such as Q and Δ are hereby confronted with a new definition following the ideas proposed in [1,2]: an Eulerian approach that focuses on manifestation of the phenomenon (kinematics) and its independence of the observer (objective). The motivation for this work is to show a useful application of vortex identification assessment in wind power projects. It is worth mentioning that, Brazil has shown a great wind energy potential and has already more than 500 wind farms [3].

Thanks to a considerable advance in computational capacity as well as the development of methods to map and measure variables – like wind speed and direction; terrain elevation; and roughness – a large amount of data can now be acquired and feed computational tools that are widely used during micrositing assessment of eolic projects.

Regarding the key aspect of any micrositing analysis: the wind resource assessment, there are two main computational methods available: the numerical implementation of simplified physics models; and Computational Fluid Dynamics (CFD) models. Simplified physics models, such as mass conservative model – OpenWind – and linearized methods – WAsP –, require less computational cost to estimate the Wind Resource Grid (WRG) as well as CFD tools, such as WindSim, computes the WRG through some Reynolds Averaged Navier-Stokes (RANS) model implementation and requires a higher computational effort. Therefore, one can notice that the core activity during wind resource assessment analysis is directly dependent on the WRG simulation and, as a result of that, this aforementioned Wind Resource Grid is a valuable and expensive data base in the wind energy market.

On the other hand, the turbulence intensity in atmospheric flows is very important for a decisive stage of a wind power project: the class selection of the wind turbines. It is well known that commercial horizontal axis wind turbines are sensitive to turbulence, impacting its mechanical efforts and aerodynamic performance. As vortex intensity is intrinsically related to turbulence, its identification and classification is important for a deeper understanding of the characteristics of atmospheric flow and for contributing to a judicious choice of wind turbines to adequately fulfill the project's lifespan.

Thus, as mentioned in the beginning of this section, the following article reports a study designed to extract unusual information from the widely used WRG data base and, consequently, applying vortex identification assessment to improve wind power project development via costless post-processing analysis of the Wind Resource Grid (WRG).

The proposed methodology of vortex identification in flows is used to refine wind resource assessment analysis calculating unusual entities alongside the standard outputted results – like pressure fields; average flow velocity fields per sector; spatial turbulence intensity distribution; and others. In the end, it is also important to note that the additional financial investment to perform this stage of post-processing calculations is low and the additional computational effort is negligible when compared to the previous calculation steps, such as wind resource simulations via CFD – the following article considers only CFD models to estimate the WRG data base and feed the developed post-processing vortex tool.

Mathematical formulation

This section presents the mathematical background used to perform the flow simulation and the post-processing stage. The subsection 2.1 presents the WindSim mathematical background, and the subsection 2.2 presents the mathematical background of the new vortex identification method. It is also important to state that WindSim was the CFD computational tool selected to estimate the WRG data base that will feed every further vortex analysis in the following work.

Flow simulation – WindSim (CFD)

WindSim solves the mean velocity and mean pressure fields using RANS equations coupled with mass and energy conservation equations, in which the k-ε model is employed to compute Boussinesq's turbulent viscosity. The system of equations which include the transport equations for k and ε are numerically solved via Finite Volume Method – utilizing PHOENICS solver.

Initial and boundary conditions are inputted by the user so WindSim calculates a timed average solution. This model exports a probabilistic distribution of wind and turbulence as a WRG data base. RANS model can be formulated as:

$$\frac{\partial U_i}{\partial x_i} = 0 \tag{1}$$

$$U_i \frac{\partial U_i}{\partial x_j} = -\frac{1}{\rho}\frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_j}\left(\upsilon \left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_i}{\partial x_i} \right) - \left(\overline{u_i u_j} \right) \right) \tag{2}$$

in which: U_i is the average velocity in i's direction; u_i is the velocity fluctuation in i's direction; x_i is the position vector component in i's direction; P is the pressure; ρ is the specific mass; and υ is the kinematic viscosity.

As mentioned before, WindSim uses k-ε model and the closure problem is treated by Boussinesq's Hypothesis (equation 3) and two differential equations artificially created to enclosure viscosity dimension.

$$\left(\overline{u_i u_j}\right) = -\upsilon_T \left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right) + \frac{2}{3}\delta_{ij}k \tag{3}$$

in which: υ_T is a dimensional proportionality coefficient called turbulent viscosity (equation 4); k is the turbulent kinetic energy; and δ_{ij} is the Kronecker Delta.

$$\upsilon_T = C_\mu \frac{k^2}{\varepsilon} \tag{4}$$

The following equations (5, 6) show those two aforementioned differential equations.

$$\frac{\partial}{\partial x_i}(U_i k) = \frac{\partial}{\partial x_i}\left(\frac{\upsilon_t}{\sigma_k}\frac{\partial k}{\partial x_i}\right) + P_k - \varepsilon \tag{5}$$

$$\frac{\partial}{\partial x_i}(U_i \varepsilon) = \frac{\partial}{\partial x_i}\left(\frac{\upsilon_t}{\sigma_\varepsilon}\frac{\partial \varepsilon}{\partial x_i}\right) + C_{\varepsilon 1}\frac{\varepsilon}{k}P_k - C_{\varepsilon 2}\frac{\varepsilon^2}{k} \tag{6}$$

in which: P_k is the turbulent kinetic energy production term (equation 7); and C_μ, σ_k, σ_ε, $C_{\varepsilon 1}$ and $C_{\varepsilon 2}$ are constants *a priori* parameterized.

$$P_k = \upsilon_t \left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right)\frac{\partial U_i}{\partial x_j} \tag{7}$$

Vortex classification

In opposition to the largely used vortex identification criteria, the addressed criterion is objective. Thus, it avoids a tricky question that cannot be easily answered by those standard criteria: which observer should be elected as the legitimate observer?

The theoretical background that supports this criterion is based on a concept stated in [4], in which an elliptical domain is defined as a region where the flow defies the tendency dictated by the symmetric part of the velocity gradient tensor ($\underline{\underline{D}}$).

The present criterion uses a different mathematical foundation to translate that concept by considering the directional tendency established by $\underline{\underline{D}}$. Therefore, the elliptical domain is defined as the region where $\underline{\underline{M}}$, the time convective covariant derivative of $\underline{\underline{D}}$, defies the directional tendency established by $\underline{\underline{D}}$.

The mathematical treatment of this concept is based on the idea that any tensor can be decomposed in two distinctive parts with respect to a symmetric tensor, namely the in-phase and the out-of-phase parts of the main tensor [1]. In the present context, this mathematical procedure is applied decoupling the tensor $\underline{\underline{M}}$ into a part that is in-phase with $\underline{\underline{D}}$ and a part that is out-of-phase with respect to $\underline{\underline{D}}$. Hence, the criterion employed in this work can spatially express domains where $\underline{\underline{M}}$ does not support the directional tendency established by $\underline{\underline{D}}$, using the information provided by the in-phase and out-of-phase parts of $\underline{\underline{M}}$ with respect to $\underline{\underline{D}}$.

The covariant convected time derivative tensorial operator $(( )^{\Delta})$ is commonly employed in the continuum mechanics literature [5]. Tensor $\underline{\underline{M}}$ is expressed by the following equation:

$$\underline{\underline{M}} = \underline{\underline{D^{\Delta}}} = \underline{\underline{\dot{D}}} + \underline{\underline{D}}\left(\underline{\underline{D}} + \underline{\underline{W}}\right) + \left(\underline{\underline{D}} - \underline{\underline{W}}\right)\underline{\underline{D}} \tag{8}$$

It is easy to demonstrate the objectivity of $\underline{\underline{M}}$

$$\underline{\underline{D^{\Delta^*}}} = \underline{\underline{Q}}\,[\underline{\underline{\dot{D}}} + \underline{\underline{D}}\left(\underline{\underline{D}} + \underline{\underline{W}}\right) + \left(\underline{\underline{D}} - \underline{\underline{W}}\right)\underline{\underline{D}}]\,\underline{\underline{Q^T}} = \underline{\underline{Q}}(\underline{\underline{D^{\Delta}}})\underline{\underline{Q^T}} \tag{9}$$

in which: $\underline{\underline{D^{\Delta^*}}}$ is the description of the tensor $\underline{\underline{M}}$ for an observer who experiences an arbitrary motion with respect to the reference observer.

The objectivity of the classifier proposed here is thus demonstrated, since such a kinematic identifier of vortices depends solely and exclusively on objective tensors – $\underline{\underline{D}}$ and $\underline{\underline{M}}$.

Finally, there is the mathematical treatment of this concept. In order to make possible an easier and more useful implementation for other areas of knowledge that investigate complex flow behaviour, it is applied the orthogonal-coaxial tensorial decomposition. In this view, the $\underline{\underline{M}}$ tensor is decoupled in coaxial and orthogonal parts in relation to $\underline{\underline{D}}$, a symmetrical tensor.

$$\underline{\underline{M}} = \underline{\underline{\phi}}_M^{\;D} + \underline{\underline{\tilde{\phi}}}_M^{\;D} \tag{10}$$

in which: $\underline{\underline{\phi}}_M^{\;D}$ is the coaxial part of the $\underline{\underline{M}}$ tensor (in phase) in relation to $\underline{\underline{D}}$; and $\underline{\underline{\tilde{\phi}}}_M^{\;D}$ is the orthogonal part of the $\underline{\underline{M}}$ tensor (out of phase) in relation to $\underline{\underline{D}}$.

Tensor $\underline{\underline{M}}$ can be re written to express the in-phase – it preserves the same eigenvectors as the reference tensor – and out of phase parts – it has orthogonality verified in relation to the reference tensor. This non-traditional expression for $\underline{\underline{M}}$ makes explicit the effective vorticity tensor $\underline{\underline{\overline{W}}}$, an entity employed by Astarita in his work [6].

$$\underline{\underline{M}} = \underline{\underline{D}}' + 2\,\underline{\underline{D^2}} + \underline{\underline{D\overline{W}}} - \underline{\underline{\overline{W}D}} \tag{11}$$

in which: $\underline{\underline{D}}'$ is the material derivative of the $\underline{\underline{D}}$ tensor keeping its eigenvectors fixed.

Entities "$\underline{\underline{D}}' + 2\,\underline{\underline{D^2}}$" (I) and "$\underline{\underline{D\overline{W}}} - \underline{\underline{\overline{W}D}}$" (II) represent orthogonal-coaxial parts of the $\underline{\underline{M}}$ tensor, when it is referenced to $\underline{\underline{D}}$. Both (I and II) are orthogonal to each other, (I) coaxial to D: preserves the inner product and (II) orthogonal to $\underline{\underline{D}}$: preserves the Lie product.

$$\underline{\underline{\tilde{\phi}}}_M^{\;D} = \underline{\underline{D\overline{W}}} - \underline{\underline{\overline{W}D}} \tag{12}$$

$$\underline{\underline{\phi}}_M^{\;D} = \underline{\underline{D}}' + 2\,\underline{\underline{D^2}} \tag{13}$$

For the task of classifying vortices in the flow, a normalized number is defined, depending on the tensors $\underline{\underline{\phi}}_M^{\ D}$ and $\underline{\underline{M}}$ so that the concept of directional corroboration to the trend dictated by $\underline{\underline{D}}$ is expressed by a number ranging from *0* to *1*. Such a classifier ($\phi_M^{\ D}$) is expressed by the following equation:

$$\phi_M^{\ D} = 1 - \frac{2}{\pi} \cos^{-1}\left(\left\| \underline{\underline{\phi}}_M^{\ D} \right\| \Big/ \left\| \underline{\underline{M}} \right\| \right) \tag{14}$$

The following table is used for the identification of the elliptical domain and, therefore, the presence and intensity of vortices:

Table 1: The criterion for vortex classification.

$\phi_M^{\ D} > 0.5$	Hyperbolic region (volume)
$\phi_M^{\ D} = 0.5$	Parabolic region (surface)
$\phi_M^{\ D} < 0.5$	Elliptical region (volume)

The vortices are assumed to pertain the elliptical region and the vortical intensity is greater as the value of the classifier $\phi_M^{\ D}$ goes to zero.

In sum, the criterion presented is objective. It is based on a solid concept that defines a vortex in a kinematic perspective and has an easy and pertinent application to most areas of science which can take advantage of complex flows deeper analysis – e.g. the study of atmospheric flow to determine a region`s wind quality for a wind resource assessment.

Methodology

Data treatment

A preliminary data treatment is necessary to generate the required information to feed the computer program, WindSim. Long-term wind speed and direction data series are obtained via linear Measure-Correlate-Predict (MCP) utilizing measurements from meteorological towers. Roughness and topography are also acquired and georeferenced.

This preliminary data treatment is crucial for the simulation, since it serves as a base for the programs to be built upon. If the inputs are not reliable the results will not be coherent. The input data consists of roughness information, digital terrain model, the wind turbine power curve – provided by wind turbine manufactures – and, finally, the location and the statistical parameters obtained through the measurements of the five anemometric towers used. Figure 1 shows a digital map terrain model and the location of the five anemometric towers.

This assemblage is utilized as an intake to calculate wind resources and energy production in the computation tool.

Numerical simulation

For the numerical simulation, the WindSim program is used with the nesting technique. Details about this technique for improving the results of the CFD program simulation are available in [7]. The nesting technique can be divided into two steps. First, it performs a simulation in a larger area with lower horizontal resolution – denominated "Large Scale" step. Then, the results of this step are used in the nesting step as initial boundary condition. In this article, the final spatial resolution is 100 m horizontally.

Post-processing stage

The post-processing stage consists in the application of the vortex identification criterion to the results of the velocity field of a typical WRG. For this purpose, a Python™ algorithm was developed in order to extract the velocity field from a WRG, and a finite difference numerical solution algorithm was implemented in the MatLab® language to apply the vortex identification criterion.

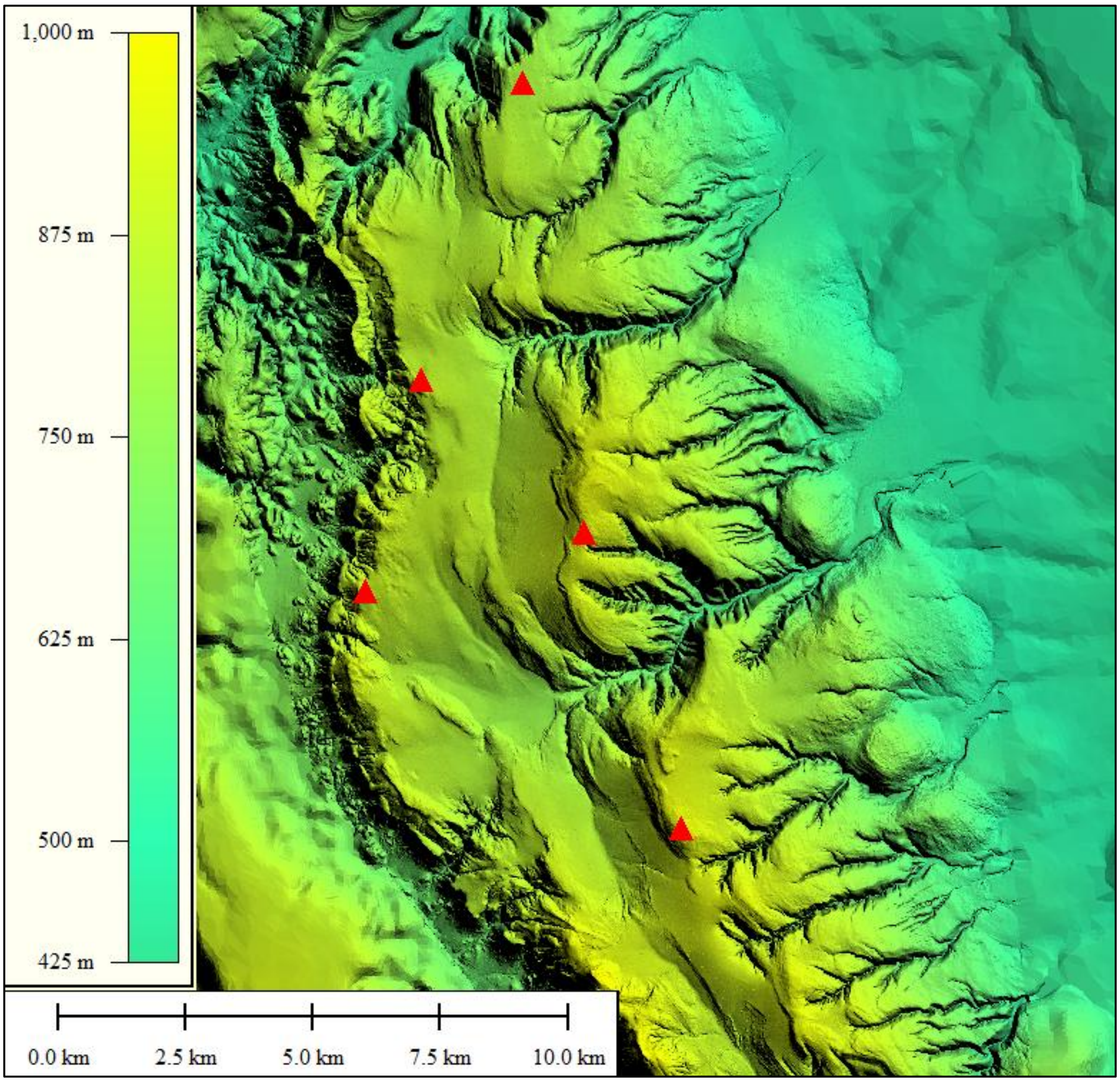

Figure 1 – Orography and meteorological masts location.

First, the necessary inputs for the MatLab® algorithm were defined: a text file containing a mesh with the components of the flow velocity field – the algorithm supports both an input of a three-dimensional field (v_x, v_y, v_z) and two-dimensional (v_x, v_y) – and the resolution information of that mesh. The mesh must be structured and regular, that is, containing a unique value for Δx_i. It is important to emphasize that the regular mesh condition does not bring any disadvantage or loss to the study of the WRG.

Once the model inputs were defined and provided to the MatLab® algorithm, the finite difference technique was used to compute the velocity gradient tensor for each point of the mesh. It was not necessary to perform a specific treatment for the domain boundaries, since the points at the edges of the velocity field mesh were excluded from the gradient analysis. This exclusion is justified by the high degree of uncertainty of the velocity field located at the edges of the domain and the small contribution it would bring to the analysis of the flow and the associated vortices. Thus, the velocity gradient tensor can be calculated point by point using the following formulation, for the two-dimensional case:

$$\frac{\partial U_i}{\partial x_i}(X,Y) = \frac{U_i(X+1,Y)-U_i(X-1,Y)}{2\Delta x_i} + O(\Delta x_i^{\,2}) \tag{15}$$

$$\frac{\partial U_i}{\partial x_j}(X,Y) = \frac{U_i(X,Y+1)-U_i(X,Y-1)}{2\Delta x_j} + O(\Delta x_j^{\,2}) \tag{16}$$

in which: the ordered pair (X,Y) represents a specific coordinate of the mesh of velocity field, so that the gradient tensor is being calculated for each point of the mesh.

After obtaining the tensor ∇U for all points of the mesh, it is easy to extract the tensors $\underline{\underline{D}}$ and $\underline{\underline{W}}$ for the entire domain – except the edge points.

The algorithm`s next step is the calculation of the material derivative of the tensor $\underline{\underline{D}}$ – this is the last necessary step for calculating the tensor $\underline{\underline{M}}$. The algorithm for the calculation of $\underline{\underline{\dot{D}}}$ is described as follows:

$$\underline{\underline{\dot{D}}} = \frac{\partial D_{pq}}{\partial t}\,\hat{e}_p\hat{e}_q + U_i\frac{\partial D_{pq}}{\partial x_i}\,\hat{e}_p\hat{e}_q \tag{17}$$

Finally, the covariant convective time derivative of the strain-rate tensor ($\underline{\underline{M}}$) is retrieved for each point of the mesh and its representation in the base of the eigenvectors of $\underline{\underline{D}}$ is calculated as following:

$$\underline{\underline{M^D}} = \underline{\underline{Q_D{}^T M Q_D}} \tag{18}$$

in which: $\underline{\underline{M^D}}$ is the tensor $\underline{\underline{M}}$ in the base of the eigenvectors of the tensor $\underline{\underline{D}}$; and $\underline{\underline{Q_D}}$ is a tensor of the eigenvectors of the tensor $\underline{\underline{D}}$.

From these entities obtained from the algorithm it is possible to recover the value calculated with the proposed classifier $\phi_M{}^D$ (Equation 14) for each point of the mesh and to identify the presence and intensity of vortices in the flow kinematic.

A validation of this methodology is presented in [8]. Such validation study is an important step to assure the quality of the result obtained when the methodology is used for the current research situation of interest: a more thorough study of the atmospheric flow.

Results

In order to evaluate the vortex identification criterion, results of the classifier $\phi_M{}^D$ obtained from WRGs at 60 m and 100 m heights are plotted.

Figures 2 to 5 present the results for the two WRGs, showing two views for each one – a two dimensional map of the vortices spatial distribution and a three-dimensional visualization map represented by a surface.

In short, by the post-processing analysis of the WRG it is possible to identify vortices formed in the atmospheric flow. As pointed out before, in section 2.2, the presence of vortices are only admitted in the elliptical region and the vortical intensity is greater as the classifier value goes to zero, that is, the more bluish area.

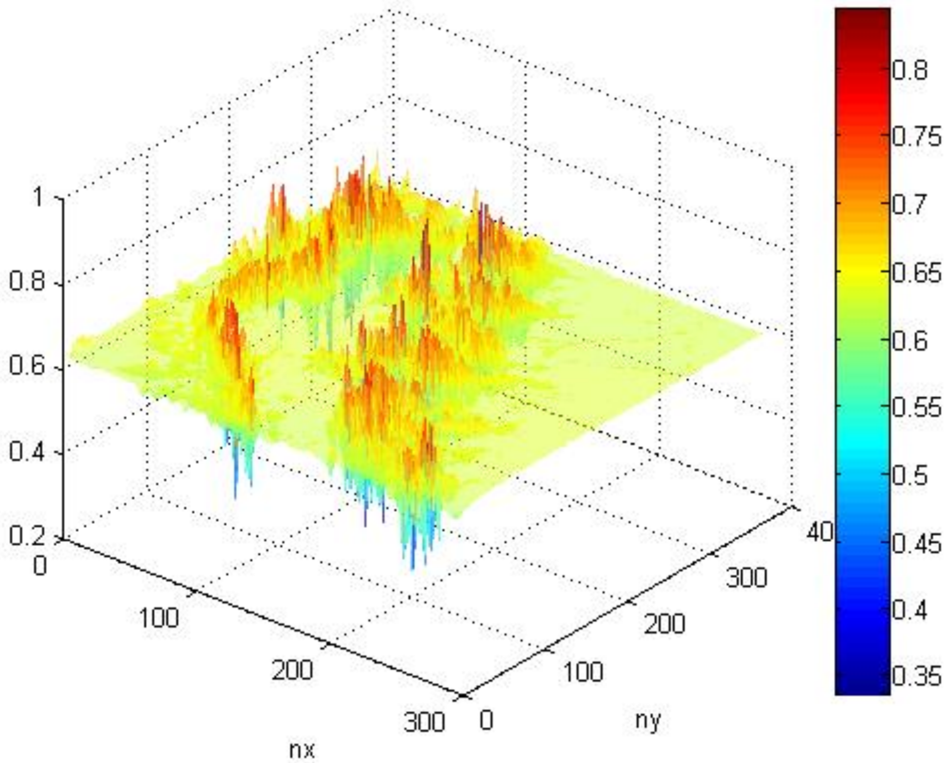

Figure 2 – Three-dimensional vortex map view at 60 m height.

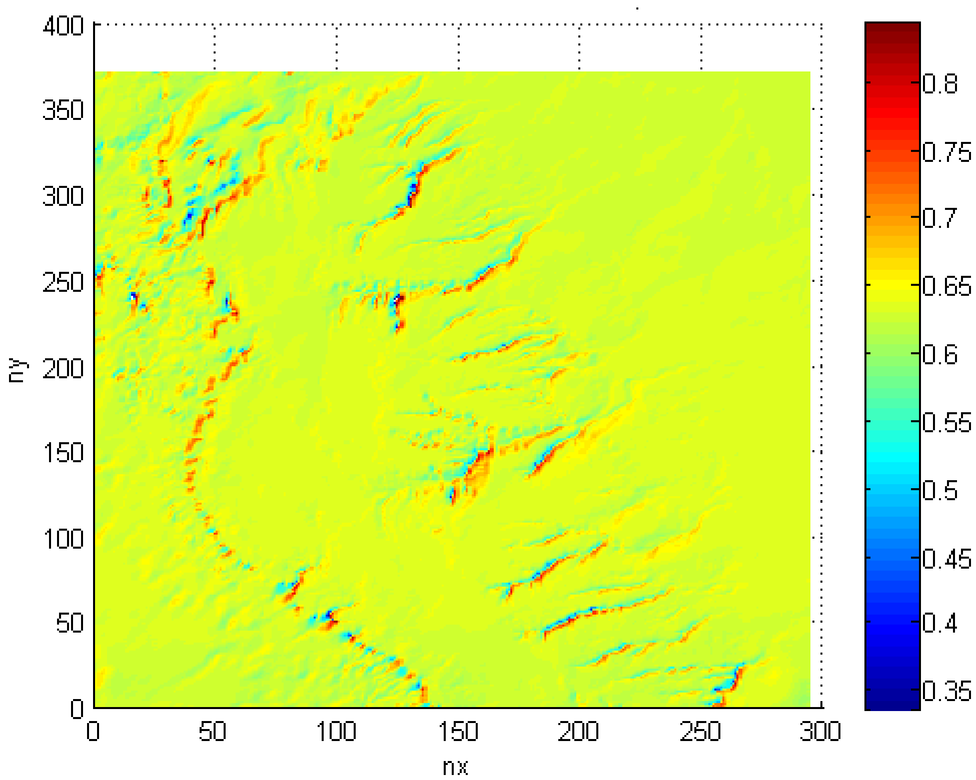

Figure 3 - Two-dimensional vortex map visualization at 60 m height.

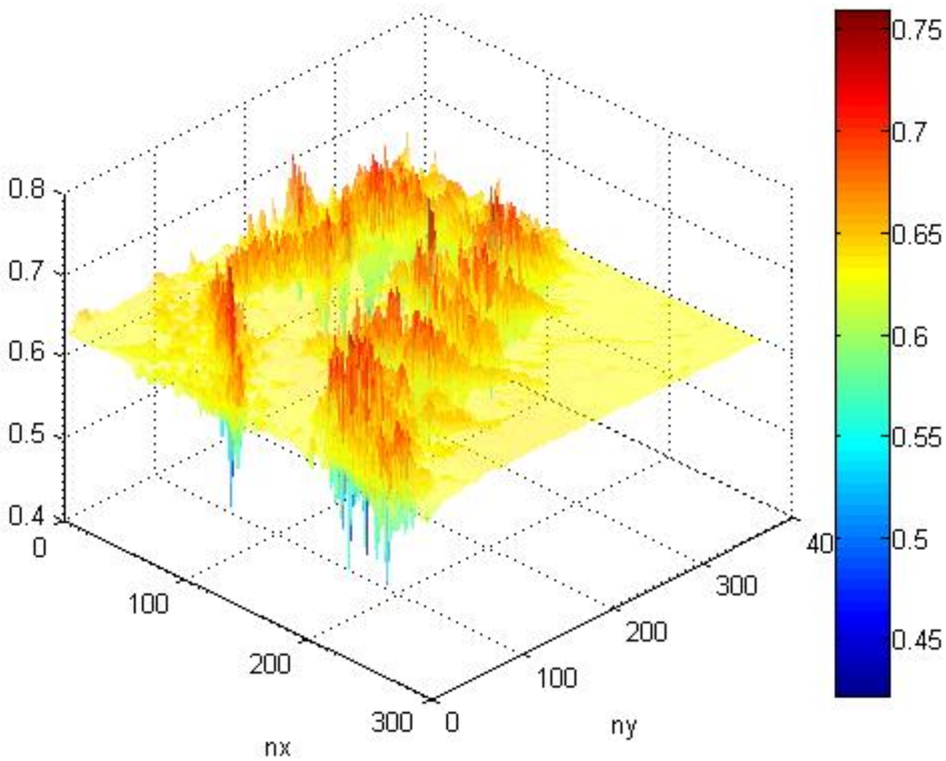

Figure 4 – Three-dimensional vortex map view at 100 m height.

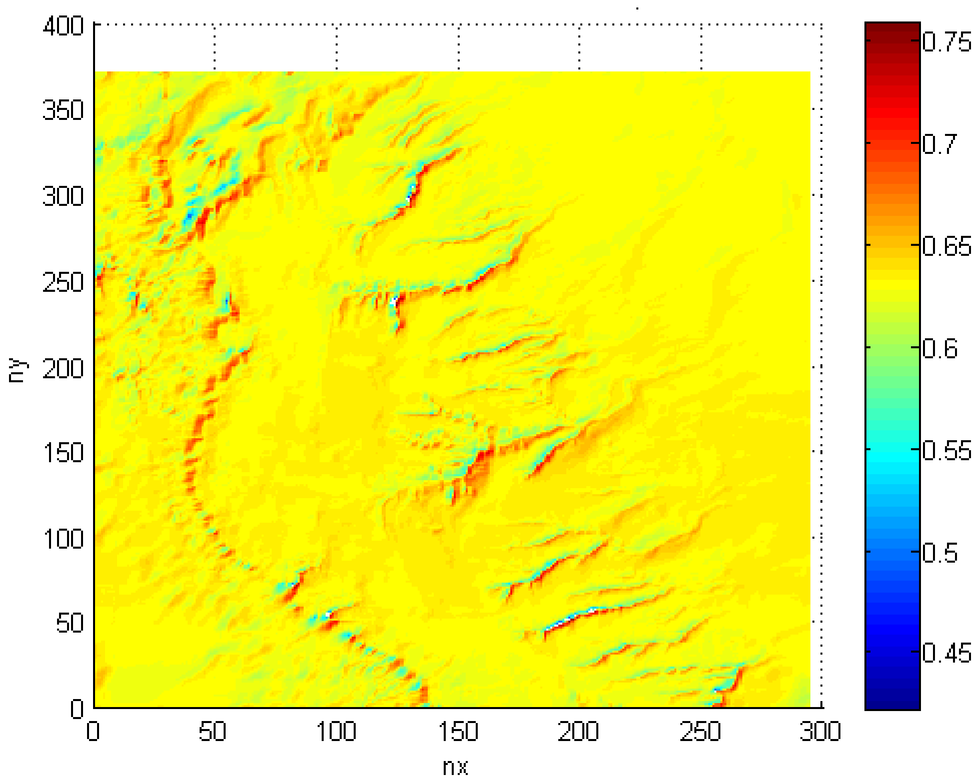

Figure 5 - Two-dimensional vortex map visualization at 100 m height.

Conclusions

First, it is important to point out that the new methodology presented here was enough to deliver further information from the widely used WRG data base.

Regarding the aforementioned results, it is possible to observe that vortice intensity increases in the lowest height of the simulated wind due to the fact that the influence of soil and vegetation is higher at lower heights. Moreover, comparing Figures 2 to 5 with the digital terrain model of Figure 1, one can see that more intense vortices are also observed within intense slope areas – areas with a rapidly and intense change of declivity.

Finally, presented results add new information to standard wind project assessments. With this approach, requirements can be better fulfilled in equipment choice, turbine layout arrangement and, consequently, diminish annual energy estimate uncertainties and losses.

Future work

The next step of this work is to apply the presented vortex identification criterion to WRGs which consider the atmospheric stability effect. For this purpose, a Bayesian inference is performed to estimate Monin-Obuckov Lenght, which will be furnished to the WindSim program. Some results of WRGs considering the atmospheric stability effect are presented in [8,9].

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